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Self similar flow behind a Shock wave

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Abstract— **Self Similar flow behind a shock wave propagating in a perfect-gas flow. Distribution of the velocities, density, Pressure components in the flow-field behind the shock are obtained.**

Keywords- Self Similar, Shockwave, perfect-gas flow

I. INTRODUCTION

Sakurai(1) has considered the problem of a shock wave arriving at the edge of a gas in a medium in which the density varies as a power law- Wang (2), Koch (3), Helliwell (4), Rayand Banerjee (5) and others have investigated the propagation of Spherical Shock waves. Singh and Srivastava (6) have considered the problem of Spherical Shock waves in on exponentially increasing medium under the material pressure. They have taken the electrical conductivity of the initial medium and the medium behind the shock to be infinite. But, in mant practical cases the medium may be of low conductivity which becomes highly conducting due to passage of a strong shock. Such a Shock wave is called a gas-ionizing shock or simply ionizing shock.

The purpose of this study are therefore, to obtain Self similar flow distribute of the velocities, density, pressure in the flow-field behind the shock are obtained. The effects of change in the index of variation of initial velocity of the medium , the index of variation of the initial density and the variation of the pressure are investigated.

II. BASIC EQUATIONS AND BOUNDARY CONDITION

The basic Eqn. governing the motion of the fluid are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial r} + \frac{2\partial \rho}{r} = 0 \;\dots\dots\dots\; (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \;\dots\dots\dots\; (2)$$

$$\frac{\partial p}{\partial r} + u\frac{\partial p}{\partial r} + pu = 0 \;\dots\dots\dots\; (3)$$

Where ρ, p are density and pressure; u is fluid velocity.

Assuming local thermodynamics equilibrium and taking Roseland's approximation, we have

$$q = -\frac{cv}{3}\frac{\partial(\sigma T^4)}{\partial x} \dots\dots\dots\dots (4)$$

Where $\frac{1}{3}\sigma C$ is the Stefan Boltzmann constant, C the velocity of light and υ the mean free path of radiation is a function of density and temperature following Wang (7),

$$\text{We take } \upsilon = \nu_0 \rho^{\alpha} T^{\beta} \ldots\ldots\ldots\ldots (5)$$

Where ν_0, α and β being constants

The total heat flux q, which appears in the energy equation may be decomposed as:

$$q = q_C + q_R \ldots\ldots\ldots\ldots (6)$$

Where q_C is the conduction heat flux, and q_R the radiation heat flux.

According to Fourier's law of heat conduction

$$q_C = -K\frac{\partial T}{\partial r} \ldots\ldots\ldots\ldots\ldots (7)$$

Where K is the coefficient of thermal conductivity of the gas and T is the absolute temperature.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning 1983), the term q_R, which represents radiative heat flux, may be obtained from the differential approximation of the radiation transport equation in the diffusion limit as:

$$q_R = -\frac{4}{3}\left(\frac{\sigma}{\alpha_R}\right)\frac{\partial T^4}{\partial r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where σ is the Stefan-Boltzmann constant and α_R in the Rosseland mean absorption coefficient

The above system of equations should be supplemented with equation of state.
A perfect gas behavior of the mediumis assumed, so that

$$p = \Gamma\rho T,\ e = \frac{\rho}{\rho(\gamma-1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9a, b)$$

Where Γ is the gas constant and γ is the ratio of specific heats.

The thermal conductivity K and the absorption coefficient α_R are assumed to vary with temperature and density. These can be written in the form of power laws, namely (Ghoniem et al. 1982)

$$K = K_0\left(\frac{T}{T_0}\right)^{\beta_c}\left(\frac{\rho}{\rho_0}\right)^{\delta_c},\quad \alpha_R = \alpha_{R_0}\left(\frac{T}{T_0}\right)^{\beta_R}\left(\frac{\rho}{\rho_0}\right)^{\delta_R},\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10a, b)$$

Where subscript '0' denotes a reference state

The exponents in the above equations should be compatible with the conditions of the problem and the form of the required solution.

We have also assumed that the total energy of the explosion is non-constant and increasing with time as (Rogers 1958; Freeman 1968; Director & Dabora 1977)

$$E = E_0 t^s \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where E_0 and s are constants. This increase of energy may be achieved by the pressure exerted on the gas by a piston. The piston may be, physically, the surface of the stellar corona or the condensed explosives or the diaphragm containing a very high pressure driver gas, at t = 0. By sudden expansion of the stellar corona or the detonation products or the driver gas into the ambient gas, a shock wave is produced in the ambient gas, in an infinitesimal time interval t_0(say). The shocked gas is separated from the expanding surface which is a contact discontinuity. This contact surface acts as a 'piston' for the shock wave in the ambient medium.

III SELF SIMILARITY TRANSFORMATIONS

We introduce the following similarity transformations to reduce the equations of motion into ordinary differential equations

$$U = \dot{R}U(\eta) \ldots\ldots\ldots\ldots (12)$$

$$\rho = \rho_0 G(\eta) \ldots\ldots\ldots\ldots (13)$$

$$p = \rho_0 P(\eta) \ldots\ldots\ldots\ldots (14)$$

Where U, G and P are functions of the non-dimensional variable η only and

$$\eta = \frac{r}{R} \ldots\ldots\ldots\ldots (15)$$

and shock velocity

$$\dot{R} = \frac{dR}{dt} \ldots\ldots\ldots\ldots (16)$$

IV SOLUTION OF EQUATIONS OF MOTIONS

In view of similarity transformation (1) to (3) the basic equations (12) to (16) take the forms

$$\eta(U-1)\dot{G} + (1+\frac{2}{\eta})GU = 0 \ldots\ldots\ldots\ldots (17)$$

$$U + (\eta + U)\dot{U} + \frac{P}{G} = 0 \ldots\ldots\ldots\ldots (18)$$

$$(U-\eta)\dot{P} + PU = 0 \ldots\ldots\ldots\ldots (19)$$

The jump conditions for a strong shock wave are

$$U_1 = \frac{2\dot{R}}{r+1} P_1 = \frac{2\rho_0}{r+1}$$

$$\rho_1 = \frac{(r-1)}{(r+1)}\rho_0$$

Where suffix denotes the values of flow variables immediately behind the shock front

The system of equation can be reduced to this equation

$$\dot{G} = \frac{-\left(1+\frac{2}{\eta}\right)GU}{(U-1)\eta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

$$\dot{U} = \frac{\left(-U-\frac{P}{G}\right)}{\eta+U} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (21)$$

$$\dot{P} = \frac{-PU}{(U-\eta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

V RESULTS

At the shock boundary we have η = 1, U = $(1 - \frac{1}{\eta})$, G = 1, P = 1

The total energy of the wave is non constant and varies as the square of the shock radius. Nature of flow variables is seen through table

η	U	G	P
1.00	1.000000	1.000000	1.000000
0.99	0.990761	1.08729	1.08739
0.98	0.982372	1.173134	1.735171
0.97	0.971471	1.265072	1.277321
0.96	0.961102	1.340721	1.312005
0.95	0.9543201	1.440322	1.463212
0.94	0.943210	1.60352	1.635098
0.93	0.937177	1.730991	1.739209
0.92	0.928719	1.845297	1.867912
0.91	0.912654	1.973292	1.913520
0.90	0.909408	2.012745	2.04832

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